Philosophy 2111: Professional Ethics Lory Lemke

Logic Primer: The Logic of Necessary and Sufficient Conditions

In general, if some p is a sufficient condition for q, then p is enough to insure that q occurs. We also know that if something is sufficient for q, it may not be also necessary for q to occur. For example, neglecting to water my indoor plants all winter is sufficient to kill them, but it is not necessary to kill them, because there are other ways of doing this dastardly job. On the other hand, giving them enough water them is not sufficient to keep them alive, but it is necessary for keeping them alive. In general, if p is a necessary condition for q, then p has to be present in order for q to occur, but it may not be sufficient for p to occur.

It is conventional in logic to use a technical term "material conditional" with strictly defined truth conditions to characterize our everyday usage of necessary and sufficient conditions. Let's take "If s, then n" to be the standard form for conditional claims, where s stands for the sufficient condition and n the necessary condition. (If you can imagine the letter to symbolize the conditional, you can use the word "SUN" to remember that in conditionals sufficient conditions come first – they are antecedents - and necessary conditions last – they are consequents. (Note: contrapositives or transpositions mean that you can switch the sufficient and necessary condition of any conditional while negating both and you will get an equivalent statement, e.g., if p, then q is equivalent to if not q, then not p.

Conditionals allow us to form some standard deductive arguments:

Modus Ponens	**Modus Tollens**
If p, then q	**If p, then q**
P	**q is false**
Hence, q	**Hence, p is false**

Occasionally, a condition will be both necessary and sufficient for another condition. We can express this as a biconditional as follows: p iff q. This says that p is both necessary and sufficient for q and vice versa. (It is simply a combination of if p, then q and if q, then p.) This amounts to saying that these two conditions are equivalent to one another, because every situation in which p is true (or false) turns out to be a situation in which q is true (or false).

Knowing this logical relationship, allows straightforward method of testing or counterexampling a biconditional. If you believe that p and q are not equivalent, which is to say that p iffq is false, then you the only way to support your belief is to find a situation where p (or q) is true (or false), while q (or p) is false (or true). If you find any of these situations, then you have found that p and q are not equivalent.