MATH 3401 (Ng/Spring 2009)
Handout on Branch-and-Bound for ILPs
March 23-25, 2009.

A summary of the Branch-and-Bound Algorithm for solving an (ILP) (max version)

Given: A max version of an (ILP)

want: an optimal solution of the (ILP) or conclude that it is infeasible or unbounded.

Initialize: Set an initial lower bound $z = -\infty$ on the objective value of (ILP). Set $i = 0$.

Step 1 (**Fathoming or bounding.**) Select LP_i, the next subproblem to be examined. Solve LP_i and attempt to fathom it using one of the three conditions.

(a) The optimal z value of LP_i cannot yield a better objective function value than the current lower bound.

(b) LP_i yields a better feasible integer solution than the current lower bound.

(c) LP_i has no feasible solution.

Two cases will arise:

(a) If LP_i is fathomed, update the lower bound if a better ILP solution is found. If all subproblems have been fathomed, STOP; the optimal ILP is associated with the current lower bound, if any. Otherwise, set $i = i + 1$, and repeat Step 1.

(b) If LP_i is not fathomed, go to Step 2 to start branching.

Step 2 (**Branching.**) Select one of the integer variables x_j, whose optimum value, x_j^* in the LP_i solution is not integer. Eliminate the region $\lfloor x_j^* \rfloor < x_j^* < \lfloor x_j^* \rfloor + 1$ (where $\lfloor x \rfloor$ denotes the largest integer $\leq x$) by creating two LP subproblems that correspond to

$$x_j \leq \lfloor x_j^* \rfloor \text{ and } x_j \geq \lfloor x_j^* \rfloor + 1$$

Set $i = i + 1$ and go to Step 1.