MATH 3401 (Ng/Spring 2009)
Handout 2
for class on **January 21-28, 2009.**

1. **(Blending Problem) Charging a Blast Furnace.**

   An iron foundry has a firm order to produce 1000 pounds of castings containing at least .45 percent of manganese and between 3.25 percent and 5.5 percent of silicon. As these particular castings are a special order, there are no suitable castings at hand. The castings sell for \$0.45 per pound. The foundry has three types of pig iron available in essentially unlimited amounts, with the following properties:

	Types of pig iron		
	A	B	C
Silicon	4%	1%	0.6%
Manganese	0.45%	0.5%	0.4%

   Further, the production process is such that pure manganese can also be added directly to the melt. The costs of the variable possible inputs are :

Pig A	\$21 per thousand pounds
Pig B	\$25 per thousand pounds
Pig C	\$15 per thousand pounds
Manganese	\$8 per pound.

   It costs 0.5 cents to melt down a pound of pig iron. Formulate an LP-model to determine the inputs that the foundry should use to produce the castings in order to maximize profits.

2. Molson Johnson has \$2200 to invest over the next five years. At the beginning of each year he can invest money in one- or two-year time deposits. The bank pays 3 percent interest on one-year time deposits and 6 percent (annually) on two-year time deposits. In addition, West World Limited will offer three-year certificates at the beginning of the third year. These certificates will return 10 percent (annually). (For the two-year and three-year investments, the assumption is that the annual interest is paid to the investor who can then choose to reinvest in any way.) If Molson reinvests his money available, including the interests he receives annually, very year, formulate a linear program to show him how to maximize his total cash on hand at the end of the fifth year.

3. Fly-R-Us *cargo plane transportation company has hired you as their* Operations Research Analyst, *and here is your chance to show them you are worth keeping.*

   The company owns several cargo airplanes; each cargo plane has *three* compartments for storing cargo, namely, *front*, *center*, and *back*. These compartments have capacity limits on both *weight* and *space*, as summarized in Table 1.

   Furthermore, the weight of the cargo in the respective compartment must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.

   There are four types of cargo and their available amounts (in weights), that have been offered to Fly-R-Us for shipment on an upcoming flight as space is available. Table 2 gives the details on the available amounts of the four types of cargo, the cargoes' attributes, and their unit profits.

   Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each cargo among the compartments of a cargo plane, to maximize total profit for the flight.

   Formulate an (LP) *model for the aforementioned problem.*

Compartments	*Weight capacity* (Tons)	*Space capacity* (Cubic feet)
Front	12	7,000
Center	18	9,000
Back	10	5,000

Table 1 : Table of capacities

Cargo types	*Amount offered* (Weight in tons)	*Volume* (Cubic feet per ton)	*Profit* $ per ton
Package	13	400	290
Chest	16	700	400
Box	20	500	320
Crate	25	600	360

Table 2 : Table of available cargo & their attributes

4. (**Transportation Problem**).

   The Brazilian Coffee Company (a.k.a. BCC) processes coffee beans into coffee at 3 plants. The coffee is then shipped every week to 7 warehouses in major cities for retail, distribution and exporting purposes. The unit shipping cost from plant i to warehouse j is c_{ij} for $i = 1, 2, 3$ and for $j = 1, 2, 3, 4, 5, 6, 7$. The production capacity of plant i is a_i for $i = 1, 2, 3$; and the demand at warehouse j is b_j for $j = 1, 2, 3, 4, 5, 6, 7$. Formulate an LP to find the production-shipping pattern in such a way that the demand at each warehouse is satisfied without violating the production capacity at each plant, and to do so at minimum total shipping cost.

5. (**A type of Scheduling Problem**.)

   The Smorris Bank is working to develop an efficient work schedule for full-time and part-time tellers. The schedule must provide for efficient operation of the bank including adequate customer service, employee breaks, and so on. On Fridays the bank is open from $9:00am$ till $7:00pm$. The number of tellers necessary to provide adequate customer service during each hour is given in **Table 3**.

Time	*Number of tellers*	*Time*	*Number of tellers*
9am-10am	6	2pm-3pm	6
10am-11am	4	3pm-4pm	4
11am-noon	8	4pm-5pm	7
noon-1pm	10	5pm-6pm	6
1pm-2pm	9	6pm-7pm	6

Table 3 : Table of Scheduled Times and Customer Service needs

Each full-time employee starts on the hour and works a 4−hour shift, followed by 1 hour for lunch and then a 3−hour shift. Part time employees work one 4−hour shift beginning on the hour. Considering salary and fringe benefits, full-time employees cost the bank $15 per hour ($105 per day) and part-time employees cost the bank $8 per hour ($32 per day).

(a) Formulate an ILP model that can be used to find an optimal schedule, i.e. a schedule that will satisfy customer service needs at a minimum total employee cost.

(b) How would you change the above formulation if the bank manager has extra requirements that there has to be at least one full-time employee on duty at all times and that there is a staff of at least five full-time employees?

6. (**A Capital Budgeting Problem.**)

The following is a general paradigm for a capital budgeting problem.

The City of Morris, Minnesota is planning its capital spending for the next T periods. There are N projects which compete for the limited capital B_i available for investment in period i. Each project requires a certain investment in each period once it is selected. Let a_{ij} be the required investment in project j for period i. The value of the project is measured in terms of the associated cash flows in each period discounted for inflation. This is called the *net present value* (NPV). Let v_j denote the NPV for project j. The problem is to select the proper projects for investments which will maximize the total value (NPV) of all the projects selected.

Here is an **instance** of the aforementioned problem. Suppose the city would like to have an expenditure plan for the next three years, i.e. there are $T = 3$ periods. Hypothetically speaking, there are 5 different projects that the city planner is looking into, for instance, a new fire engine or a community center etc.., i.e. $N = 5$. **Table 4** gives the appropriate values for the required investments (in thousands of dollars), the available funds, and also the individual project's NPV or returns.

Set up an ILP model that can be used to solve this problem.

	Expenditures for:			
Project	Year 1	Year 2	Year 3	Returns or NPV
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Available funds	25	25	25	

Table 4 : Data for an instance of capital budgeting problem

7. (**Set Covering Problem.**) A firm has four possible sites for locating its warehouses. The cost of locating a warehouse at site i is $\$K_i$. There are nine retail outlets, each of which must be supplied by at least one warehouse. It is not possible for any one site to supply all the retail outlets as shown in **Figure 1**.

The firm needs to decide which sites to build its warehouses so that all its retail outlets can be supplied to and to do so at minimum total cost.

Set up an (ILP) model that can be used to solve this problem.

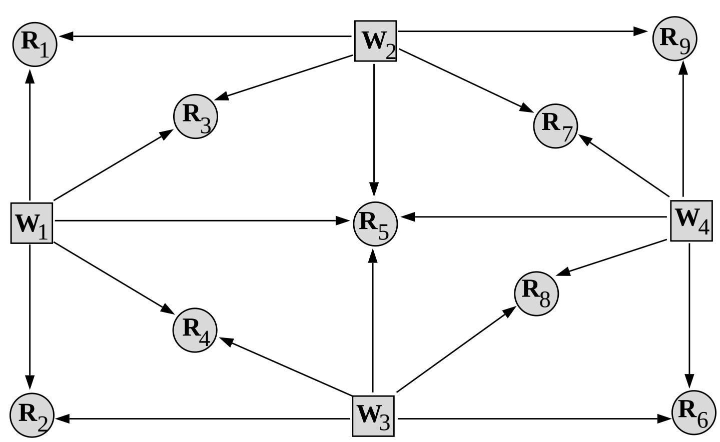

Figure 1 : Set-covering problem possibilities

8. (**Warehouse Location Problem.**) A retail firm is planning to expand its activities in an area by opening two new warehouses. Three possible sites are under consideration as shown in **Figure 2**. Four customers have to be supplied whose demands are D_1, D_2, D_3, and D_4, respectively.

   Assume any two sites can supply all the demands but site 1 can supply customers 1, 2, and 4 only; site 3 can supply customers 2, 3, and 4; while site 2 can supply all the customers. The unit transportation cost from site i to customer j is c_{ij}. For each warehouse site we have the following data:

Site	Capacity	Initial Capital Investment ($)	Fixed Operating Cost ($)
1	A_1	K_1	P_1
2	A_2	K_2	P_2
3	A_3	K_3	P_3

   The optimization problem is to select the proper sites for the two warehouses which will minimize the total costs of investment, operation, and transportation. Set up an *MILP* model for this problem.

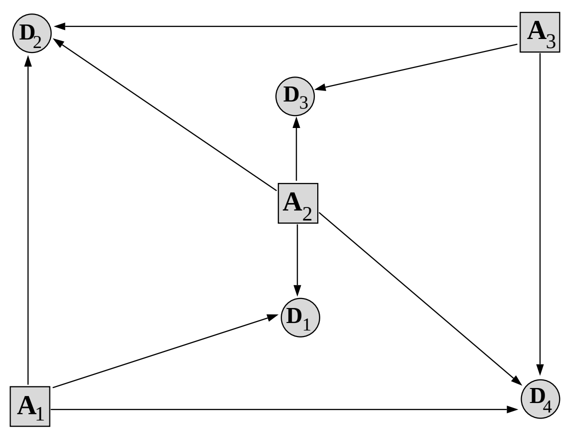

Figure 2 : Warehouse location problem