MATH 3401 (Ng/Spring 2009)
Assignment 1
Due Friday, January 30, 2009.

1. (20pts). The *Giant Mismanagement Corporation* (a.k.a. GMC) has three plants with excess production capacity. All three plants have the capability of producing a certain product, and management has decided to use one of the excess production capacity in this way. This product can be made in three sizes – large, medium, and small – that yield a net unit profit of \$140, \$120, and \$100, respectively. Plants 1, 2, and 3 have the excess manpower and equipment capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. However, the amount of available in-process storage space also imposes a limitation on the amount of production. Plants 1, 2, and 3 have 13000, 12000 and 5000 square feet of excess in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.
   Sales forecasts indicate that 900, 1200, and 750 are the maximum number of units of the large, medium, and small sizes, respectively, that can be sold per day.
   To maintain a uniform work load among the plants and to retain some flexibility, GMC has decided that the additional production assigned to each plant must use the same percentage of the excess manpower and equipment capacity.
   GMC wishes to know how many units of each of the sizes should be produced by each of the plants to maximize total profit from this product, and it (GMC) has hired you to do this by formulating the problem as an LP-model.

2. (20pts.) Bubba and Bubbette Johnson procreated eight years ago. In anticipation of the immense college expenses for Bubba Jr., they decided to start an annual investment program on the child's eighth birthday (now) that will last until the eighteenth birthday. Judging from Bubba and Bubbette's financial position over the next 10 years, the couple estimates that they will be able to invest the following amounts at the beginning of each year:

Year	1	2	3	4	5	6	7	8	9	10
Amount (\$)	2000	2000	2500	2500	3000	3500	3500	4000	4000	5000

   To avoid unpleasant surprises, Bubba and Bubbette opt to invest the money very safely. The following options are open to them:

   (a) Insured savings with 5.5% annual yield.
   (b) Six-year government bonds that yield 6.5% annually and have a current market price equal to 0.96 of the face value.
   (c) Nine-year municipal bonds yielding 7.2% annually and having a current market price equal to 1.03 of the face value.

   Bubba and Bubbette would like to know how they should invest their money over the next 10 years so that their total yield at the end of the tenth year will be maximized. Formulate the couple's problem as an LP model; you are not asked to solve it.

   (F.Y.I., market price of bonds is the amount of money the investor actually pays to buy the bonds. Each bond yields simple annual interest on its face value, and its face value is surrendered at maturity date. In addition, the annual interest paid for any of the types of investments are reinvested into any of the three types of investments at any year.)

3. (20pts.) The school board of the Smorris Area Schools has made the decision to close down the Crack Road Junior High middle school (sixth, seventh, and eighth grades) at the end of this school year and reassign all of next year's middle school students to the three remaining middle schools, namely, Smorris Junior High, Northside Junior High, and Gundown Road Junior High.

   The school district provides bussing for all middle school students who must travel more than approximately a mile, so the school board needs a plan for reassigning the students so that their total bussing cost is minimized. The cost per student of bussing from each of the six residential areas of the city to each of the schools is shown in Table 1 along with other basic data for next year. (0 *means no bussing is needed, and* – *means that particular assignment is not possible.*)

					Bussing Cost Per student		
Area	*Number of Students*	*% 6th Grade*	*% 7th Grade*	*% 8th Grade*	*Smorris Jr. High*	*Northside Jr. High*	*Gundown Jr. High*
1	450	32	38	30	3	0	7
2	600	37	28	35	0	4	5
3	550	31	32	37	6	3	2
4	350	28	39	33	2	5	–
5	500	39	34	27	0	–	0
6	450	33	29	38	5	3	4
			School capacity:		900	1,100	1,000

Table 1 : Table of Data for Bussing Middle Grade Students

   The school board also has imposed the restriction that each grade must constitute between 30 and 35 percent of each school's population. Table 1 shows the percentage of each area's middle school population for next year that falls into each of the three grades. The school attendance zone boundaries can be drawn so as to split any given area among more than one school, but assume that the percentages shown in Table 1 will continue to hold for any partial assignment of an area to a school.

   (a) (17pts.) Formulate a linear programming model for determining how many students should be assigned from each area to each of the three schools so that the frugal City of Smorris could spend the least amount of money on bussing middle grade students.

   (b) (3pts.) It is conceivable that an optimal solution to your problem in part (a) could contain fractional components. Modify the formulation in part (a) to make the solution more realistic. (If correct, this new formulation is called an *integer* linear program).

4. (20pts.) Farmer Jack wishes to determine the best selection of stock for his farm, his objective being to maximize his profit after the sale of the animals at the end of the period. The alternatives available are *Merino* sheep, *Romney* sheep, *Southdown* sheep, *Hereford* cattle and *Jersey* cattle. Jack has calculated that each Merino would require one acre of land, and would cost \$1.50 in extra feed, treatment, etc. The purchase price is \$160 and Jack estimates that the selling price at the end of the period will be \$300. The following table gives the respective data for the different types of the stock.

	Land req.	Unit cost in feed	Purchase price	Selling price
Merino	1 acre	\$1.50	\$160	\$300
Romneys	1 acre	\$1.75	\$125	\$250
Southdowns	1 acre	\$1	\$100	\$200
Herefords	4 acres	\$15	\$130	\$210
Jerseys	6 acres	\$12	\$180	\$320

The size of the farm is $1,000$ acres, and the farmer has $\$15,000$ with which to purchase and to maintain the stock.

(a) (17pts.) Formulate the problem as a linear program.

(b) (3pts.) It is conceivable that an optimal solution to your problem in part (a) could contain fractional components. Embellish the formulation in part (a) to make the solution more realistic. (If correct, this new formulation is called an *integer* linear program).

5. (20pts.) An agricultural mill manufactures feed for chickens. This is done by mixing several ingredients, such as corn, limestone, or alfalfa. The mixing is to be done in such a way that the feed meets certain levels for different types of nutrients, such as protein, calcium, carbohydrates, and vitamins. To be more specific, let $R = \{1, 2, 3 \ldots, n\}$ be the set of ingredients, i.e. there is a total of n ingredients; and let $S = \{1, 2, 3, \ldots, m\}$ be the set of m nutrients. Let the cost of ingredient j be c_j, for $j = 1, 2, \ldots, n$. The amount of feed needed is b, and we know that a_{ij} is the amount of nutrient i present in a unit of ingredient j, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. It is also known that the acceptable lower and upper limits of nutrient i in a unit of the chicken feed are l_i and u_i, respectively. Finally, due to shortages, the mill cannot acquire more than t_j units of ingredient j. Formulate a linear program to determine the amount of ingredients used in the feed, and to do so at minimum cost.

6. (20pts.) *University of Mighty Mouse* (a.k.a. UMM)'s Vice-president for planning has compiled a list of badly needed construction projects $P = \{p : p = 1, 2, \ldots, N\}$. For each project $p \in P$, he has an estimate of the cost c_{yp} of the project in its 1st ($y = 1$), 2nd($y = 2$), ... year, once it has begun. Some projects are also interrelated. Project pairs $(p, q) \in M$ are mutually exclusive i.e. at most one of p or q can be undertaken, where M is a collection of size 2 subsets of P. Project pairs $(p, q) \in D$ are dependent, i.e. p can be undertaken only if q was started in a prior year, where D is another collection of size 2 subsets of P. Naturally, there is a limited budget, b_f, in each fiscal year f's budget improvement, for $f = 1, 2, \ldots, F$. Like all university administrators, the VP sees his role as making life as easy as possible for UMM's administrators. He has estimated for each project, p, a convenience (or **sucking-up**) coefficient v_p, measuring how much the project will be appreciated by his administrative colleagues. Formulate, as a pure binary integer linear programming model, the problem of choosing deciding which project to choose and which fiscal year the chosen project should start, in order to conform to budgets, M, and D, while maximizing total amount of convenience.