

Math 3401 (Ng/Spring 2009)
Assignment 2
Due Friday February 6, 2009.

1. (50pts). (For each of the following problems, please address the specific issues/questions below).

- Determine if the following (*LP*)'s are feasible, infeasible, or unbounded by using the graphical interpretation method.
- Sketch the graphs of the constraints; indicate the feasible region, objective contours and the objective direction.
- If the (*LP*) is optimal, then find an optimal solution point and find the optimal value.

a.

$$\begin{aligned} \text{Maximize } z &= 30x_1 + 20x_2 \\ \text{s.t.} \\ -x_1 + 2x_2 &\leq 10 & (1) \\ -x_1 - x_2 &\geq -8 & (2) \\ 5x_1 + 3x_2 &\leq 30 & (3) \\ x_1 &\geq 0 & (4) \\ x_2 &\geq 0 & (5) \end{aligned}$$

b.

$$\begin{aligned} \text{Minimize } z &= 3x_1 - 6x_2 \\ \text{s.t.} \\ 5x_1 + 7x_2 &\leq 35 & (1) \\ -x_1 + 2x_2 &\leq 2 & (2) \\ x_1 &\geq 0 & (3) \\ x_2 &\geq 0 & (4) \end{aligned}$$

c.

$$\begin{aligned} \text{Minimize } z &= -2x_1 + 3x_2 \\ \text{s.t.} \\ -x_1 + 2x_2 &\leq 2 & (1) \\ 2x_1 - x_2 &\leq 3 & (2) \\ x_2 &\geq 4 & (3) \\ x_1 &\geq 0 & (4) \\ x_2 &\geq 0 & (5) \end{aligned}$$

d.

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 2x_2 \\ \text{s.t.} \\ 5x_1 + 5x_2 &\geq 25 & (1) \\ -4x_1 + 4x_2 &\leq 12 & (2) \\ 2x_1 - 6x_2 &\leq 24 & (3) \\ x_1 &\geq 0 & (4) \\ x_2 &\geq 0 & (5) \end{aligned}$$

e.

$$\begin{aligned}
 \text{Minimize } z &= x_1 + 3x_2 \\
 \text{s.t.} & \\
 5x_1 + 5x_2 &\leq 25 & (1) \\
 -4x_1 + 4x_2 &\leq 12 & (2) \\
 x_1 + 2x_2 &\geq 12 & (3) \\
 x_1 &\geq 0 & (4) \\
 x_2 &\geq 0 & (5)
 \end{aligned}$$

2. (30pts). For each of the following (*LP*)'s,

- write it in its canonical form
- identify the basic variables, the non-basic variables, and a basic feasible solution from each of the canonical forms.

(You are NOT asked to solve these problems to optimality.)

a.

$$\begin{aligned}
 \text{Minimize } z &= 13x_1 - 12x_2 - 7x_3 + 19x_4 \\
 \text{s.t.} & \\
 -9x_1 - x_2 + 3x_3 + x_4 &\geq -12 & (1) \\
 2x_1 - 5x_3 + 9x_4 &\leq 7 & (2) \\
 x_1 + 5x_3 - 6x_4 &\leq 9 & (3) \\
 x_1, x_2, x_3, x_4 &\geq 0
 \end{aligned}$$

b.

$$\begin{aligned}
 \text{Minimize } z &= -13x_1 + 12x_2 \\
 \text{s.t.} & \\
 3x_1 - 5x_2 + x_3 &\geq -7 & (1) \\
 6x_1 + 7x_2 &\leq 4 & (2) \\
 x_1 \text{ unrestricted, } x_2 \leq 0, x_3 \geq 0
 \end{aligned}$$

c.

$$\begin{aligned}
 \text{Maximize } z &= 5x_1 + 3x_2 - 2x_3 \\
 \text{s.t.} & \\
 -7x_1 + 5x_2 - x_3 &\leq 6 & (1) \\
 3x_1 + 4x_2 + 6x_3 + x_4 &\geq 12 & (2) \\
 x_1 - 5x_2 &\geq -6 & (3) \\
 x_1, x_2, x_3, x_4 &\text{ unrestricted}
 \end{aligned}$$