MATH 3401 (Ng/Spring 2009)
Assignment 3
Due February 13, 2009.

Note: For problems that require the simplex method, use the dictionary format.

1. (10pts). Use the *Simplex Algorithm* to show **and** to **justify clearly** why the following (LP) is unbounded. (Also, explain in words, why you are convinced that the problem is unbounded).

$$\begin{aligned} \text{Maximize } z &= 3x_1 + x_2 \\ \text{s.t.} & \\ x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 &= 2 && (1) \\ x_1 + \frac{1}{2}x_3 - \frac{1}{2}x_4 &= 1 && (2) \\ -\frac{3}{2}x_3 + \frac{1}{2}x_4 &\leq 2 && (3) \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

2. (20pts). Consider the following (LP) :

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 5x_2 - 4x_3 \\ \text{s.t.} & \\ 3x_1 + 4x_2 + 2x_3 &\leq 16 && (1) \\ 4x_1 + 6x_2 + 2x_3 &\leq 10 && (2) \\ 12x_1 + 3x_2 + 6x_3 &\leq 33 && (3) \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

   a. Find an initial basic feasible solution with slack variables as the basic variables.
   b. Use the primal simplex method to improve the solution in part (a.). (One step).
   c. Identify the new basic and non-basic variables after part (b.)
   d. Use Primal Simplex method to solve the problem.

3. (20pts). Use the **Big-M** method to solve the following (LP)'s.

   a.

$$\begin{aligned} \text{Maximize } z &= 4x_1 - 5x_2 - 3x_3 \\ \text{s.t.} & \\ x_1 + x_2 + x_3 + x_4 &= 11 && (1) \\ x_1 + x_2 &\geq 1 && (2) \\ 2x_1 + 3x_2 + x_3 &\geq 20 && (3) \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

b.

$$\begin{aligned}
\text{Maximize}\, z &= x_1 - 2x_2 - 4x_3 + 2x_4 \\
\text{s.t.} & \\
3x_1 + 2x_2 - x_3 - 2x_4 &\leq -3 \qquad (1) \\
-x_1 + x_2 - 2x_3 - x_4 &\geq 10 \qquad (2) \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

4. (20pts). Solve the following (LP)'s by the **Two-Phase** method.

a.

$$\begin{aligned}
\text{Minimize}\, z &= -2x_1 + 2x_2 + x_3 + x_4 \\
\text{s.t.} & \\
x_1 + 2x_2 + x_3 + x_4 &\leq 2 \qquad (1) \\
x_1 - x_2 + x_3 + 5x_4 + x_5 &\geq 4 \qquad (2) \\
2x_1 - x_2 + x_3 &\geq 2 \qquad (3) \\
x_1, x_2, x_3, x_4, x_5 &\geq 0
\end{aligned}$$

b.

$$\begin{aligned}
\text{Maximize}\, z &= x_1 - 2x_2 - 4x_3 + 2x_4 \\
\text{s.t.} & \\
3x_1 + 2x_2 - x_3 - 2x_4 &\leq -3 \qquad (1) \\
-x_1 + x_2 - 2x_3 - x_4 &\geq 10 \qquad (2) \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

5. Use **TORA** (on any computers in the lab in Sci 2530) or **CPLEX** (on my Unix machine) to **verify** your answers to the problems above.

   TORA is easier to use and more user-friendly, but it has a limit to the number of variables (eg 50-60) or the number of constraints it can handle.

   CPLEX is really the Simplex Method in C - it is an industrial or research software; it was made to handle large-scale (LP)s and (ILP)s. (If you wish to use CPLEX, please see me for an account on my machine.)

   For problems with a small number of variables and constraints, I recommend TORA. For research problems, I recommend CPLEX.