Math 3401 (Ng/Spring 2009)
Assignment 4
Due Friday, February 27, 2009

1. (30pts). Consider the following General Linear Program (P).

$$\text{Maximize } z = \sum_{j=1}^{Q} \alpha_j X_j + \sum_{k=1}^{\Gamma} \beta_k T_k + \sum_{l=1}^{\Phi} \gamma_l W_l$$

Subject to:

$$\begin{array}{rcll}
\displaystyle\sum_{j=1}^{Q} a_{ij} X_j + \sum_{k=1}^{\Gamma} e_{ik} T_k + \sum_{l=1}^{\Phi} g_{il} W_l & = & b_i & \text{for } i = 1, 2, \ldots, \Theta \\
\displaystyle\sum_{j=1}^{Q} h_{mj} X_j + \sum_{k=1}^{\Gamma} s_{mk} T_k + \sum_{l=1}^{\Phi} \mu_{ml} W_l & \leq & v_m & \text{for } m = 1, 2, \ldots, \Pi \\
X_j & \geq & 0 & \text{for } j = 1, 2, \ldots, Q \\
T_k & \geq & 0 & \text{for } k = 1, 2, \ldots, \Gamma \\
W_l & \geq & 0 & \text{for } l = 1, 2, \ldots, \Phi
\end{array}$$

where the decision variables are X_j for $j = 1, 2, \ldots, Q$; T_k for $k = 1, 2, \ldots, \Gamma$; and W_l for $l = 1, 2, \ldots, \Phi$.

a. How many variables does the primal problem (P) have?
b. How many constraints does the primal problem have?
c. How many variables does the dual problem (D) have?
d. How many constraints does the dual problem have?
e. What is the objective function of the dual problem?
f. What is the last main dual constraint?
g. What is the constraint numbered $(Q + \Gamma + 2)$ in the dual problem of (P)?
h. What is the constraint numbered $\Theta + \Pi$ in the dual of the dual problem of (P)?
i. How many dual variables are *unrestricted* variables?
j. Are the first Θ dual variables going to be variables which are ≥ 0 or ≤ 0 or *unrestricted*?

2. (20pts). Consider the following primal problem (P).

$$\text{Minimize } z = 24x_1 + 20x_2 + 48x_3 + 25x_5$$

subject to:

$$\begin{array}{rrrrrcl}
x_1 & +x_2 & +3x_3 & -3x_4 & +x_5 & \geq & 2 \\
2x_1 & +x_2 & 2x_3 & +2x_4 & & \geq & 2 \\
x_1, & x_2, & x_3, & x_4, & x_5 & \geq & 0
\end{array}$$

(a) Write down the dual of (P).
(b) Solve the dual problem by using TORA. Write down your optimal solution point to the dual problem.
(c) Utilize the information about the dual problem and its solution, and all the Duality results to solve the primal problem (P). (Do **NOT** solve (P) using the *Simplex* or TORA).

3. (20pts). Consider the following primal problem (P).

$$\begin{array}{lrrrcr} \text{Maximize } z = & 3x_1 & +1x_2 & +2x_3 & & \\ \text{subject to:} & & & & & \\ & x_1 & +3x_2 & +2x_3 & \geq & 10 \\ & 6x_1 & +2x_2 & +x_3 & \leq & 30 \\ & x_1 & +x_2 & +x_3 & = & 5 \\ & x_1, & x_2, & x_3 & \geq & 0 \end{array}$$

Without using the *Simplex algorithm or TORA*, and by using the Complementary Slackness Properties, determine if

a. $(1, 4, 0)$ is an optimal solution to the primal problem (P),

b. $(-1, 0, 4)$ is an optimal solution of (P)'s dual problem (D).

4. (20pts). Consider the following primal problem (P).

$$\begin{array}{lrrcl} \text{Maximize } z = & -x_1 & +x_2 & & \\ \text{subject to:} & & & & \\ & x_1 & -x_2 & \leq & 0 \\ & x_1 & & \leq & 5 \\ & x_1, & x_2 & & \textit{unrestricted} \end{array}$$

(a) Solve the above problem (P) graphically or by using TORA.

(b) From your answer in part(a.), what can you say about the dual problem (D)?

(c) Solve the dual problem (D) graphically, or by using TORA.

5. In problems 2 and 4 above, if you choose to use TORA, please enclose a printout with your answers.