Math 3401 (Ng/Spring 2009)
Assignment 5
Due Friday, March 13, 2009.

1. (10pts). Consider the following Linear Program (P).

$$\begin{array}{lrrrrcl} \text{Maximize } z = & -5x_1 & -3x_2 & -3x_3 & -6x_4 & & \\ \text{subject to:} & & & & & & \\ & -6x_1 & +x_2 & +2x_3 & +4x_4 & \leq & 14 \\ & 3x_1 & -2x_2 & -x_3 & -5x_4 & \leq & -25 \\ & -2x_1 & +x_2 & & +2x_4 & \leq & 14 \\ & & & & x_j & \geq & 0 \text{ for } j = 1,2,3,4 \end{array}$$

Using slack variables for all the constraints above as an initial basis, solve (P) using the *Dual Simplex* method.

2. (20pts). Consider the following Linear Program (P).

$$\begin{array}{lrrrrcl} \text{Maximize } z = & 2x_1 & +4x_2 & +4x_3 & -3x_4 & & \\ \text{subject to:} & & & & & & \\ & x_1 & +x_2 & +x_3 & & = & 4 \\ & x_1 & +4x_2 & & +x_4 & = & 8 \\ & & & & x_j & \geq & 0 \text{ for } j = 1,2,3,4 \end{array}$$

Suppose the following dictionary is optimal for (P), and you should verify the optimal value using Tora.

$$\begin{array}{rcrrr} x_2 & = & 2 & -\frac{1}{4}x_1 & -\frac{1}{4}x_4 \\ x_3 & = & 2 & -\frac{3}{4}x_1 & +\frac{1}{4}x_4 \\ \hline z & = & 16 & -2x_1 & -3x_4 \end{array}$$

Now, we would like to alter the objective function of (P), and find its new optimal solution without resolving the entire linear programming problem from scratch.

For each of the following objective functions, find the new optimum solution to the new (P) by using the sensitive analysis procedure.

a. Minimize $z = 4x_1 - 3x_2 - 4x_3 + x_4$.

b. Maximize $z = 2x_1 + 5x_2 + 2x_3 + 4x_4$

3. (30pts). Consider a product-mix problem in which each of three products is processed on three different operations. The limits on the available time for the three operations are 430, 460, and 420 minutes daily and the profits per unit of the three products are \$3, \$2, and \$5. The times in minutes per unit on the three operations are given as follows:

	Product 1	Product 2	Product 3
Operation 1	1	2	1
Operation 2	3	0	2
Operation 3	1	4	0

The LP model (P), is written as :

$$\begin{array}{lrrrcll} \text{Maximize } z = & 3x_1 & +2x_2 & +5x_3 & & & \text{(daily profit)} \\ \text{subject to:} & & & & & & \\ \text{(operation 1):} & x_1 & +2x_2 & +x_3 & \leq & 430 & \\ \text{(operation 2):} & 3x_1 & +0x_2 & +2x_3 & \leq & 460 & \\ \text{(operation 3):} & x_1 & +4x_2 & +0x_3 & \leq & 420 & \\ & & & x_j & \geq & 0 & \text{for } j = 1,2,3 \end{array}$$

where x_1, x_2, x_3 denote the amount of product $1, 2, 3$ produced, respectively; and x_4, x_5, x_6 are the appropriate slack variables for the aforementioned LP.

a. Fill in the objective function row in the dictionary given below. Is this dictionary an optimal one for (P)? **Explain**.

$$\begin{array}{rcrrrr} x_2 & = & 100 & +\frac{1}{4}x_1 & -\frac{1}{2}x_4 & +\frac{1}{4}x_5 \\ x_3 & = & 230 & -\frac{3}{2}x_1 & & -\frac{1}{2}x_5 \\ x_6 & = & 20 & -2x_1 & +2x_4 & -x_5 \\ \hline z & = & & & & \\ \hline \end{array}$$

b. Determine the new optimal solutions if the time limits (in minutes) of the daily usages of the operations are changed to:

i.

$$\begin{bmatrix} 420 \\ 460 \\ 440 \end{bmatrix}$$

ii.

$$\begin{bmatrix} 300 \\ 800 \\ 200 \end{bmatrix}$$

c. Suppose we add a fourth operation to all the products of the aforementioned original LP problem. The times in minutes per unit on the fourth operation are 4, 1, 2 for products 1, 2, 3, respectively. Determine the new optimal solution assuming that the daily capacity of the fourth operation is limited by:

i. 570 minutes.

ii. 548 minutes.

4. (30pts). Consider the LP model (P) for the product-mix problem in the previous question. Find the new optimum solution for each of the following simultaneous changes:

   a. Objective function : $z = 5x_1 + 3x_2 + 4x_3$
   Right-hand side: $(420, 460, 440)$

   b. Objective function : $z = 4x_1 + 3x_2 + 2x_3$
   Right-hand side: $(300, 800, 600)$

   c. Objective function : $z = 2x_1 + x_2 + 3x_3$
   Right-hand side: $(300, 400, 150)$

5. (20pts). The Concrete Products Corporation has the capability of producing four types of concrete blocks. Each block must be subjected to four processes: batch mixing, mold vibrating, inspection, and yard drying. The plant manager would like to maximize profit for next month. During the upcoming thirty days, he has 800 machine hours available on the batch mixer, 1000 hours on the mold vibrator, and 340 man-hours of inspection time. Yard-drying time is unconstrained. The production manager has formulated his problem as a linear program (max version) with the initial dictionary:

$$\begin{array}{rcllllll}
x_5 &=& 800 & -x_1 & -2x_2 & -10x_3 & -16x_4 & \text{(batch mixing)} \\
x_6 &=& 1000 & -1.5x_1 & -2x_2 & -4x_3 & -5x_4 & \text{(mold vibrating)} \\
x_7 &=& 340 & -0.5x_1 & -0.6x_2 & -x_3 & -2x_4 & \text{(inspection)} \\
\hline
z &=& & 8x_1 & +14x_2 & +30x_3 & +50x_4 &
\end{array}$$

where x_1, x_2, x_3, x_4 represent the number of pallets of the four types of blocks. After solving by the simplex method, the optimal dictionary is:

$$\begin{array}{rcllllll}
x_2 &=& 200 & -11x_3 & -19x_4 & -1.5x_5 & +x_6 \\
x_1 &=& 400 & +12x_3 & +22x_4 & +2x_5 & -2x_6 \\
x_7 &=& 20 & -0.4x_3 & -1.6x_4 & -0.1x_5 & +0.4x_6 \\
\hline
z &=& 6000 & -28x_3 & -40x_4 & -5x_5 & -2x_6
\end{array}$$

   a. By how much must the profit on a pallet of number 3 blocks be increased before it would be profitable to manufacture them? (Profitable to manufacture blocks of type 3 means you want x_3 to become basic so that its value could be positive).

   b. What minimum profit on x_2 must be realized so that it remains in the production schedule i.e. so that the current x_2 remains non-zero in optimal solution?

   c. If the 800 machine-hours capacity on the batch-mixer is uncertain, for what range of machine hours will it remain feasible to produce blocks of types 1 and 2?

   d. A competitor located next door has offered the manager additional batch-mixing time at a rate of \$6.00 per hour. Should he accept it? Justify your answer. (Hint: Recall that the shadow price or the dual price of first constraint is the change in optimal value per unit increase of batch-mixing hour, within appropriate range. And the value of this shadow price is the optimal dual value corresponding to the (batch-mixing) constraint).