Math 3401 (Ng/Spring 2009)
Assignment 6
Due date: Friday April 3, 2009.

1. Three company trucks must be assigned to pick up 7 miscellaneous loads on the way back (i.e. as backhauls) from their regular deliveries. Truck capacities and load sizes are indicated in the table below, together with the extra distance each truck would have to travel if it is to deviate to pick up a load.

load number	1	2	3	4	5	6	7	
load size	4	8	13	31	11	9	21	truck capacity
for truck 0	23	25	29	12	49	37	2	30
for truck 1	45	72	13	23	7	39	9	40
for truck 2	50	23	41	40	42	59	20	50

   (a) (10pts.) Formulate the (Generalized Assignment) problem of finding a minimum total extra distance assignment of these loads to trucks. You should have one binary variable for each truck-load combination, one inequality capacity constraint for each truck, and one equality main constraint for each load (i.e. every load must be picked up).

   (**Note:** Please VERIFY your formulation in Prob 1(a) with me before you start applying B-B; and please do this by Wed Mar 25.)

   (b) (10pts.) Solve the above problem by the *branch-and-bound* algorithm. Do it interactively, i.e. use (TORA) to solve the *LP*-**relaxations**. Show the branch-and-bound tree and show the order in which you solved the LP-relaxations of the candidate problems. And you **must** adhere to the following rules when branching:

   (i.) Branch on the fractional variable closest to 1.000.

   (ii.) Use depth-first enumeration with $= 1$ preferred to $= 0$ in the case of ties.

   (c) (10pts.) The equality constraints of this particular model pose natural alternatives to our usual branching scheme (branch $x = 1$, $x = 0$). Devise a branching rule that would produce three candidates (a.k.a. children ☺) at each branch rather than two; illustrate what this branching rule looks like. Show that (as required for convergence of algorithm) every solution is in at least one of the newly created candidates. Can a solution be in more than one candidate region under your rule?

2. (30pts.) Solve the following problems by the *branch-and-bound* algorithm. Please feel free to use TORA to solve all the required *LP*-relaxation problems. You are required to show the branch-and-bound tree.

   a.

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 2x_2 \\ \text{s.t.} & \\ 2x_1 + 2x_2 &\leq 9 && (1) \\ 3x_1 + 3x_2 &\leq 18 && (2) \\ x_1 &\geq 0 \text{ and integer} && (3) \\ x_2 &\geq 0 \text{ and integer} && (4) \end{aligned}$$

b.

$$
\begin{aligned}
\text{Maximize } z &= 2x_1 + 3x_2 \\
\text{s.t.} & \\
5x_1 + 7x_2 &\leq 35 && (1)\\
4x_1 + 9x_2 &\leq 36 && (2)\\
x_1 &\geq 0 \text{ and integer} && (3)\\
x_2 &\geq 0 \text{ and integer} && (4)
\end{aligned}
$$

c.

$$
\begin{aligned}
\text{Maximize } z &= x_1 + x_2 \\
\text{s.t.} & \\
2x_1 + 5x_2 &\leq 16 && (1)\\
6x_1 + 5x_2 &\leq 30 && (2)\\
x_1 &\geq 0 \text{ and integer} && (3)\\
x_2 &\geq 0 \text{ and integer} && (4)
\end{aligned}
$$

3. (10pts.) Solve the following (ILP) using the *cutting-planes* algorithm for pure integer programming problem. Use TORA to get the optimal solution to the initial LP-relaxation of (ILP).

$$
\begin{aligned}
\text{Maximize } z &= 4x_1 + 6x_2 + 2x_3 \\
\text{s.t.} & \\
4x_1 - 4x_2 &\leq 5 && (1)\\
-x_1 + 6x_2 &\leq 5 && (2)\\
-x_1 + x_2 + x_3 &\leq 5 && (3)\\
x_1 &\geq 0 \text{ and integer} && (4)\\
x_2 &\geq 0 \text{ and integer} && (5)\\
x_3 &\geq 0 \text{ and integer} && (6)
\end{aligned}
$$

4. (10pts.) Solve the following mixed-integer linear programming problem $(MILP)$ using the *cutting-planes* algorithm for mixed-integer programming problem. Use TORA to get the optimal solution to the initial LP-relaxation of (ILP).

$$
\begin{aligned}
\text{Maximize } z &= 4x_1 + 6x_2 + 2x_3 \\
\text{s.t.} & \\
4x_1 - 4x_2 &\leq 5 && (1)\\
-x_1 + 6x_2 &\leq 5 && (2)\\
-x_1 + x_2 + x_3 &\leq 5 && (3)\\
x_1 &\geq 0 \text{ and integer} && (4)\\
x_2 &\geq 0 && (5)\\
x_3 &\geq 0 \text{ and integer} && (6)
\end{aligned}
$$