

Math 3401 (Ng/Spring 2009)
Assignment 8
Due date: Wednesday April 29, 2009.

1. (10pts.) Consider the directed graph $G = (V, A)$ in Figure 1, with net supplies $b_1 = 2$, $b_2 = 20$, $b_5 = -10$, $b_7 = -10$, $b_3 = b_4 = b_6 = 0$. The numbers on the arcs are unit costs of flows.

Prove that the corresponding Minimum Cost Network Flow problem has no feasible solution.

(Hint: you may prove this by contradiction.)

2. (70pts.) Now, consider a new graph obtained from G in Figure 1 by adding a new vertex called 8 and adding new arcs called $(1, 8)$ and $(2, 8)$ with unit costs, $c_{1,8} = 0$, $c_{2,8} = 0$ and net supply $b_8 = -2$.

Let us call this new graph, \tilde{G} .

- The subgraph of \tilde{G} induced by $\{(2, 1), (2, 4), (4, 6), (5, 4), (6, 7), (4, 3), (1, 8)\}$ is a spanning tree of \tilde{G} . Exhibit this spanning tree of \tilde{G} .
- Compute the corresponding primal basic and (complementary) dual solutions on your spanning tree in part (a). Use vertex 7 as your first vertex when computing primal solution, and start the dual with $u_7 = 0$.
- Was the primal solution of part (b) primal feasible? Was the dual solution of part (b) dual feasible? Explain why or why not.
- Replace arc $(5, 4)$ in the (basis) spanning tree by the (non-basic i.e.) non-spanning tree arc $(3, 5)$, and repeat parts (b) and (c).
- Compute the (adjusted) or reduced costs $\bar{c}_{ij} \triangleq c_{ij} - u_i + u_j$ from your current basic solution in part (d). Show that the corresponding linear programming solution is **not** optimal and that the arc $(5, 7)$ can enter the basis.
- Assuming that $(5, 7)$ enters the basis, determine what basic arcs' flows would be impacted, **and** by how much per unit of x_{57} .
- Use the results of part (f) to determine which variable would leave the basis if $(5, 7)$ entered in the primal network simplex algorithm. Then show the revised (basis) spanning tree that would result and verify its primal feasibility by updating the primal solution.

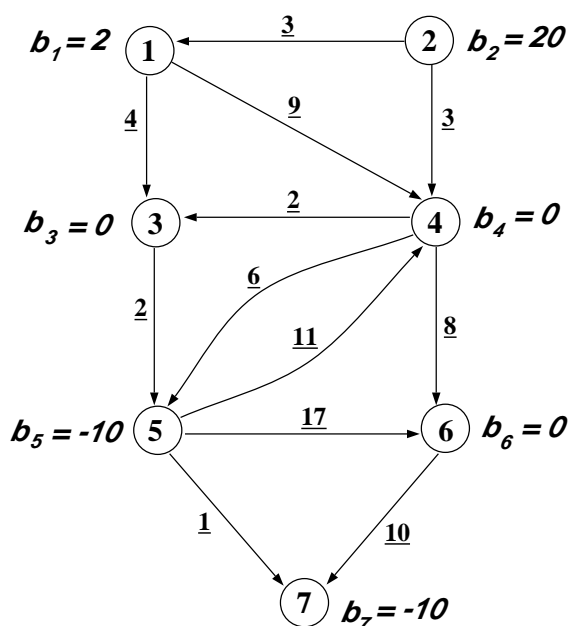


Figure 1 : Graph, $G = (V, A)$ for Prob 1