Math 3401 (Ng/Spring 2009)
Network Simplex Handout
April 20 - 27, 2009.

Big Picture of Network Simplex Algorithm for $(MCNF)$ problem

a. Compute a primal basic solution and find an initial basic feasible solution, if one exists.

b. Compute a (complementary) dual solution using the current primal basic solution.

(There is a dual variable u_i for each vertex $i \in V$).

c. Compute (adjusted) or reduced costs

$$\bar{c}_{ij} = c_{ij} - u_i + u_j$$

i.e. reduced costs for arcs (i, j) is the original costs c_{ij} − dual value of tail i + dual value of head j.

(Note that by virtue of part(b), the reduced costs of basic arcs are always 0).

d. **Check optimality**. If $\bar{c}_{ij} \geq 0$ for all arcs then the current flow (i.e. basic feasible solution) is optimal; STOP.

Otherwise there exists a non-basic arc (i, j) whose reduced cost $\bar{c}_{ij} < 0$. Choose this arc to enter the basis.

e. **Update or detect unboundedness**. When a non-basic arc enters, it creates a **unique** cycle together with the spanning tree. Choose the arc on the cycle whose flow will be forced to 0 when the non-basic arc enters at some flow value $\theta > 0$. This particular arc will leave the basis; update the flow and repeat (b).

If no such arc in the cycle exists, i.e. there is no arc in the cycle whose flow will go to 0 as θ increases indefinitely, then the problem is unbounded; STOP.