MATH 3411 (Ng/Fall 2009)
A few applications of Discrete Dynamical Systems
for class on **October 22, 2009.**

1. **A competition for weapons of mass destruction.**

   Suppose there are two countries, Almeda and Bonobo, which are extremely competitive in acquiring weapons in the name of *defence*. Let a_n and b_n represent the amount (in US dollars) spent on weapons in year n by Almeda and Bonobo, respectively. Assumptions of the model are:

   - each country has some fixed amount of distrust of the other, causing it to retain weapons,
   - if Bonobo spends a lot of money on defence in any year, then Almeda spends more money the next year,
   - similarly, if Almeda spends a lot of money on defence in any year, then Bonobo spends more money the next year,
   - a large expenditure on weapons by each country in one year will cause smaller expenditure in weapons in that country in the following year. (Maybe this is because large expenditure in one year can deplete a country's reserve in its treasury!)

   Write down a DDS with more than one equation to describe the amount of money spent on weapons by Almeda and Bonobo.

2. **Discrete predator-prey model.**

   Deep in the redwood forests of California,dusky-footed wood rats provide up to 80% of the diet for the spotted owl, the main predator of the wood rat. Let O_n and R_n be the population of owl and wood rats, respectively, after n years.

   Possible assumptions of the model are:

   - the higher the population of owls is in one year, the lower the population of rats is in the next year, whereas
   - the higher the population of rats is in one year, the higher the population of owls is in the next year,
   - for either species, their population in one year is also directly proportional to its population in the previous year,

   Write down a DDS with more than one equation to describe the population of owls and wood rats after n years.

3. **Pollution in a connected Lake System**

   (This came from a project funded by the U of M's **U**ndergraduate **R**esearch **O**pportunities **P**rogram (UROP) and completed by Jeanna Schultz (UMM 2003) and faculty advisor P.N.)

   Although it is more common to study pollution levels of a self-enclosed system such as one lake, by and large, lakes that are in close proximity do affect one another in terms of pollution levels.

   For instance, in nearby Meeker County, there is a lake system with at least five lakes in near proximity or even connected with small rivers. Let's consider only two of them, say, Lake Manuella and Lake Stella. Suppose a_n and b_n represent the amount of pollution level in week n in Lake Manuella and Lake Stella, respectively.

   Possible assumptions of the model are:

   - the amount of pollution in Lake Manuella at any week depends on the amount of pollution it has the previous week, the amount of pollution that flows into it from Lake Stella and the amount of pollution that comes from an external source (eg trash and junk caused by humans),
   - the amount of pollution in Lake Stella at any week depends on the amount of pollution it has the previous week, the amount of pollution that flows into it from Lake Manuella and the amount of pollution that comes from an external source (eg trash and junk caused by humans),
   - the rate of flow of pollution from Manuella to Stella need not be the same as that from Stella to Manuella due to different currents in the connecting river.

   Write down a DDS with more than one equation to describe the amount of pollution in Lake Manuella and Lake Stella after n weeks.