

MATH 3411 (Ng/Fall 2009)
Handout 1 (Mathematical Induction: from text and elsewhere)
September 8 - 10, 2009.

1. In each of the following problems, use induction to prove that each equation is true for every positive integer n .

(a)

$$1 + 3 + 5 + \dots + (2n + 1) = (n + 1)^2$$

(b)

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(c)

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots + (-1)^{n+1} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}$$

(d)

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

2. Let T_n be the following recursively defined sequence.

$$T_1 = 1, T_2 = 2; \quad T_k = T_{k-1} + T_{k-2} \text{ for } k \geq 3$$

Show, by mathematical induction, that T_n , in a non-recursive form, can be written as:

$$T_n = \frac{1}{\sqrt{5}} \left\{ \left(\frac{1}{2} + \frac{1}{2}\sqrt{5} \right)^{n+1} - \left(\frac{1}{2} - \frac{1}{2}\sqrt{5} \right)^{n+1} \right\} \text{ for } n = 1, 2, 3, \dots$$

3. Use mathematical induction to verify each of the following inequalities.

(a)

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

whenever n is a positive integer greater than 1.

(b)

$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \text{ for } n = 1, 2, \dots$$

(c)

$$2^n \geq n^2; \text{ for } n = 4, 5, 6, \dots$$

- (d) Suppose for any positive integer, k ,

$$H_k = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}$$

Show that

$$H_{2^n} \geq 1 + \frac{n}{2} \text{ for } n = 0, 1, 2, 3, \dots$$

(Note: For those of you who have taken Calculus 2 or who are familiar with series, you may recognize H_k as the **partial sum of the Harmonic** series. Having proved the aforementioned inequality, you can also provide an alternative proof of why the *Harmonic* series diverges.

4. By experimenting with small values of n , guess a formula for the given sum. Then use induction to prove that your formula is unequivocally correct for any positive integer n .

(a)

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)}$$

(b)

$$4 \left(\frac{1}{2 \cdot 3} \right) + 8 \left(\frac{2}{3 \cdot 4} \right) + 16 \left(\frac{3}{4 \cdot 5} \right) + \dots + 2^{n+1} \left[\frac{n}{(n+1)(n+2)} \right]$$