MATH 3411 (Ng/Fall 2009)
Handout 3 - examples of counting process
for class on **October 27 - November 3, 2009**

1. How many different choices of dinners can a person get at Ron's Greasy Spoon Restaurant? (Assume that a dinner means one appetizer, a main course, and a beverage.)

2. Consider the letters: $A \quad B \quad C \quad D \quad E$.

   (a) How many strings of length 4 can be formed using the above letters if repetitions are NOT allowed?

   (b) How many of those strings in part (a) begin with the letter B?

   (c) How many of those strings in part (a) do NOT begin with the letter B?

3. How many $8-$bit strings have exactly two 1's?

4. In how many ways can we select two books of different subjects from among 8 distinct computer science books, 5 distinct math books, and 3 distinct art books?

5. A six-person committee made up of Peter, Paul, Mary, Larry, Mo, and Curly, is to select a *chairperson*, a *secretary*, and a *treasurer*.

   (a) In how many ways can this be done?

   (b) In how many ways can this be done if Mo must hold one of the officers?

   (c) In how many ways can this be done if Peter and Mary must hold one of the above positions?

   (d) In how many ways can this be done if Peter or Mary must be the chairperson?

6. How many permutations of the letters: $A \quad B \quad C \quad D \quad E$ contain the substring DEF?

7. How many permutations of the letters: $A \quad B \quad C \quad D \quad E$ contain a substring DEF where the letters D, E and F in the substring can be in any order?

8. In how many ways can 6 people be seated around a circular table?

9. In how many ways can we select a chairperson, vice-chairperson, secretary, and treasurer from a group of 10 people?

10. In how many ways can seven distinct Martians and five Klingons wait in line if no two Klingons stand together?

11. A group of eighteen students, Nicole, Tara, Ryan, Dan, Joey, Wenting, Chris, Jeff, Robert, Casey, Missy, Kevin, Lynnette, Jodi, Justin, Cong, Lucas, and Danielle decided to talk with the chair of the *Division of Science & Mathematics* about having the division offer more topics courses in discrete mathematics. The division chair has said that he will speak with 5 of the students.

    In how many ways can these 18 students choose five of their group to talk with the division chair?

12. An ordinary deck of 52 cards consists of four suits (*clubs*, *diamonds*, *hearts*, and *spades*) of 13 denominations each (*ace*, $2 \to 10$, *jack*, *queen*, *king*).

    (a) How many (unordered) five-card poker hands, selected from an ordinary $52-$card deck, are there?

    (b) How many poker hands contain cards all of which are of the same suit?

    (c) How many poker hands contain three cards of one denomination and two cards of another denomination?

13. A *Genetic Code Problem.*

    The genetic code of organisms is stored in DNA and RNA molecules as a long string of four nucleotides commonly represented (in RNA) by the letters A, C, G, and U. Short strings of RNA can be *biologically sequenced* by biotech methods. Although the RNA sequence for a single gene has hundreds of thousands of letters, there exists special enzymes that will split a long string into short fragments (which can be sequenced) by breaking the string immediately following each appearance of a particular letter.

    Suppose a C-enzyme (which splits after each appearance of a C) breaks a $20-$letter string into 8 fragments, which are identified as:

    $$AC\,,\ AC\,,\ AAAUC\,,\ C\,,\ C\,,\ C\,,\ UAUA\,,\ UGGC$$

    Note that each fragment except the last one on the string, must end with a C. How many different strings could have given rise to this set of fragments?

14. In how many ways can 8 distinct books be divided among three students if *Bill* received four books, and *Sam* and *Mary* received two books each.

15. Consider three books: a *combinatorics* book, a *physics* book, and a *history* book. Suppose that the library has at least six copies of each of these books. In how many ways can we select six books?

16. In how many ways can 12 identical mathematics books be distributed among the students Michaelangelo, Leonardo, Raphael, and Donatello?

17. Suppose that there are piles of red, blue, and green balls and that each pile contains at least eight balls.

    (a) In how many ways can we select eight balls?

    (b) In how many ways can we select eight balls if we must have at least one ball of each color?

18. Consider the following equation, where x_i are non-negative integers for $i = 1, 2, 3, 4$.

    $$x_1 + x_2 + x_3 + x_4 = 29$$

    (a) How many solutions are there to the aforementioned equation?

    (b) How many solutions are there to the aforementioned equation, that satisfy $x_1 > 0$, $x_2 > 0$, $x_3 > 0$, $x_4 \geq 0$?