

MATH 3411 (Ng/Fall 2009)
Handout 4-more on counting process & HW
October 27 - November 3, 2009.

1. The options available on a particular model of a car are five interior colors, six exterior colors, two types of seats, three types of engines, and three types of stereos. How many different possibilities are available to the consumer?
2. How many eight-bit strings read the same from either end? (*An example of such an eight-bit string is 01111110. Such strings are called palindromes.*)
3. A ten-person committee composed of Mary, Samuel, Joshua, Timothy, Luke, Matthew, Ruth, Naomi, Jonathan, and David is to select a *chairperson*, *secretary*, and *treasurer*.
 - (a) How many selections exclude Jonathan?
 - (b) How many selections are there in which Mary is an officer and Luke is not?
 - (c) How many selections are there in which Naomi is either chairperson or treasurer?
4. In how many ways can five distinct Martians and eight distinct Klingons be seated at a circular table?
5. A club consists of 6 distinct men and 7 distinct women.
 - (a) In how many ways can we select a committee of four persons that has at most one man?
 - (b) In how many ways can we select a committee of four persons that has persons of both sexes?
6. In how many ways can we select a committee of four Romulans, three Vulcans, and two Cardassians from a group of 10 distinct Romulans, 12 distinct Vulcans, 4 distinct Cardassians?
7. How many eight-bit strings contain at least two 0's in a row?
8. Find the number of (unordered) five-card poker hands, selected from an ordinary 52-card deck, having the properties indicated.
 - (a) Containing four aces
 - (b) Containing cards of exactly two suits.
9. Suppose we have a shipment of 50 microprocessors of which four are defective.
 - (a) In how many ways can we select a set of four microprocessors?
 - (b) In how many ways can we select a set of four microprocessors containing at least one defective microprocessor?
10. Determine the number of strings that can be formed by ordering the letters given in:
 - (a) *GUIDE*
 - (b) *SCHOOL*
 - (c) *SALESPERSONS*
11. In how many ways can 10 distinct books be divided among three students if the first gets five books, the second gets three books, and the third gets two books?
12. Suppose we have piles of identical red, blue, and green balls where each pile contains at least 10 balls.

- (a) In how many ways can 10 balls be selected?
- (b) In how many ways can 10 balls be selected if exactly *one* red ball must be selected?
- (c) In how many ways can 10 balls be selected if twice as many red balls as green balls must be selected?

13. Find the number of non-negative integer solutions of the equation

$$x_1 + x_2 + x_3 = 15$$

subject to the conditions given.

- (a) $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.
- (b) $x_1 \geq 0, x_2 > 0, x_3 = 1$.

14. Find the number of non-negative integer solutions to:

$$x_1 + x_2 + x_3 + x_4 = 12$$

satisfying $0 \leq x_1 \leq 4, 0 \leq x_2 \leq 5, 0 \leq x_3 \leq 8, 0 \leq x_4 \leq 9$.

15. Thirteen people have first names Gabrielle, Xena and Athena, and last names Ng, Clinton, Rosenthal and McDermott. Show that at least two persons have the same first and last names.
16. Is it possible to interconnect five processors so that exactly two processors are directly connected to an identical number of processors? **Explain.**
17. An inventory consists of a list of 80 items, each marked “available” or “unavailable”. There are 50 available items. Show that there are at least two unavailable items in the list either three or six items apart.
18. A 3×7 rectangle is divided into 21 squares each of which is colored red or black. Prove that the board contains a *nontrivial rectangle* (not $1 \times k$ or $k \times 1$) whose four corner squares are all black or all red.
19. My date and I went to a party with 10 other couples. Several people shook hands, but no one shook hands more than once with the same person and no one shook their dates’ hands or their own hands.
I asked everyone (including myself) at the party, except my date, how many hands they had shaken and discovered none of the 21 had shaken the same number of hands. How many hands did my date shake? **Justify your answer.**
20. Thirteen people have first names Leonardo, Donatello, Michaelangelo and Raphael, and last names McDermott, O’Loughlin and Ng. Show that at least two people have the same first and last names.
21. If 20 processors are possibly interconnected, show that at least two processors are directly connected to the same number of processors.
22. An inventory consists of a list of 80 items, each marked *available* or *unavailable* but not both. There are 45 available items. Show that there are at least two available items in the list and they are exactly nine items apart. (For instance, available items at positions 13 & 22 or 69 & 78 would satisfy the condition).
23. A useful feature of black-and-white pictures is the average brightness of the picture. Let us say that two pictures are similar if their average brightness differs by no more than some fixed value. Show that among six pictures, there are either three that are mutually similar or there are three that are mutually dissimilar.