MATH 3411 (Ng/Fall 2009)
Assignment for undirected and directed graphs anytime on November 12, 2009.
(Please slide it under my office door.)

1. (10pts.) From section 6.5 on graph isomorphism in Handout 5 (on Fundamentals of Graph Theory), you were asked to identify which graphs G_i is isomorphic to one another.

   Prove that the graphs G_2 is isomorphic to the graph in G_6. (You need to clearly label all vertices and edges, define the functions $\theta : V(G_2) \longrightarrow V(G_6)$ and $\phi : E(G_2) \longrightarrow E(G_6)$ such that $e = (u, v) \in E(G_2)$ if and only if $\phi(e) = (\theta(u), \theta(v))$, and show that these functions are bijections.)

2. (10pts) Show that *connectivity* in a given undirected graph $G = (V, E)$ is an equivalence relation on the set of vertices V.

   (Write up the proof rigorously; I have already given you the main ideas in class.)

3. (10pts). Let $T = (V, E)$ be a non-trivial tree, i.e. $E \neq \emptyset$.

   (a) Prove that there exists a unique path in T between any two vertices $u, v \in V$.
   (Hint: First prove **existence**, and then prove that the path is **unique**.)

   (b) Prove that there is at least one vertex $u \in V$ such that $d_T(u) = 1$.
   (Hint: It may be easier to prove this by contradiction.)

4. (10pts). Let $T = (V, E)$ be a tree. Use induction on $|V|$ to show that the number of vertices is one more than the number of edges, i.e.

$$|V| = |E| + 1$$

   (Write up the proof rigorously; I have already given you the main ideas in class.)
   (Hint: your work may be easier if you use the results you proved in the previous problem.)

5. (20pts.) Let $G = (V, E)$ and $H = (\bar{V}, \bar{E})$ be the graphs given in **Figure 1**.

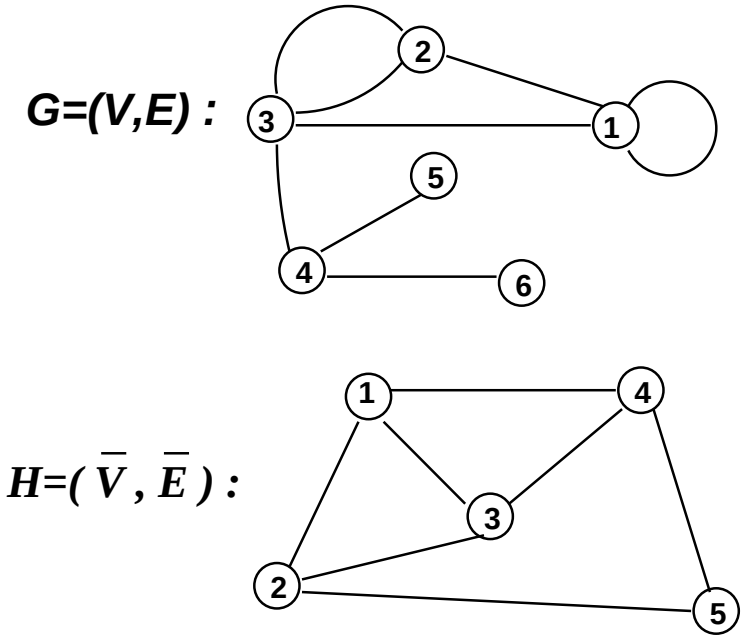

Figure 1 : Graphs, $G = (V, E)$ and $H = (\bar{V}, \bar{E})$

(a) Find the vertex-edge incidence matrices of G and H; identify them clearly and identify the rows and columns clearly.

(b) Find the adjacency matrices of G and H; identify them clearly and identify the rows and columns clearly.

6. (10pts.) Let $G = (V, A)$ be the directed graph given in **Figure 2**. Find its vertex-arc incidence matrix; identify the rows and columns clearly.

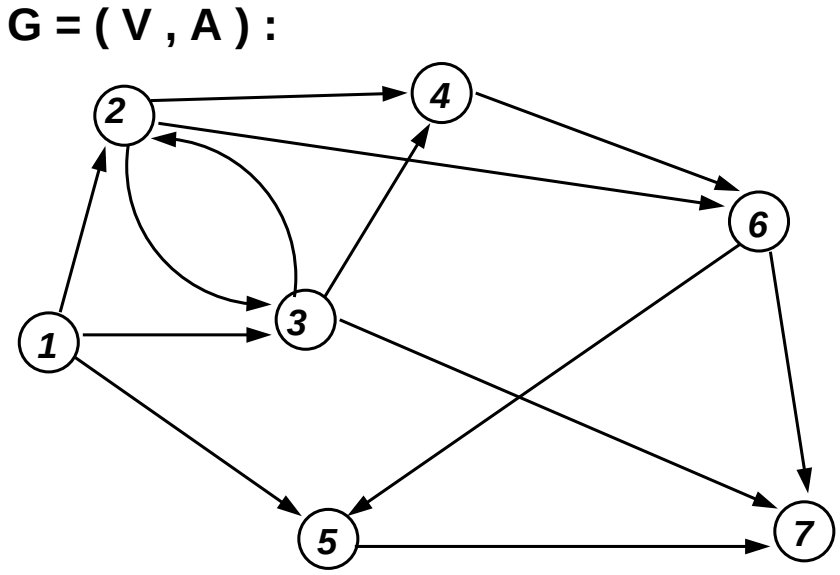

Figure 2 : Directed graph, $G = (V, A)$